THE RELIABILITY ANALYSIS OF THE TEETH OF EVOLVENT GEARINGS WITHIN MECHANICAL GEAR BOXES

Eugenia SECARĂ
Transilvania University of Brasov, secarae@unitbv.ro

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Abstract

The evolvent gearings compose subsystems of the mechanical gear boxes. Therefore, the reliability of the mechanical gear boxes depends on the reliability of the teeth within evolvent gearings. Considering as known the g_F percentage reliability the paper determines the t_h operating period of time.

The reliability of a system represents the probability that after a certain operating period in given conditions the system parameters do not exceed the limits that were set. To solve this problem the relation between the system parameters and the parameters of the system elements has to be known.

The evolvent gearings form different subsystems within the mechanical gear boxes. Thus the reliability of the mechanical gear boxes also depends on the reliability of the teeth of the evolvent gearings.

The a_f statistic safety parameter of the teeth is defined by the following relation:

$$a_F = \frac{\overline{\sigma_{Flim}} - \overline{\sigma_F}}{\left(D_{\sigma_{Flim}} + D_{\sigma_r}\right)^{\frac{1}{2}}} \tag{1}$$

where: $\overline{s_F}$ represents the medium effective tension at the base of the tooth;

$\overline{s_{F\lim}}$ represents the limit tension specific to tiredness due to bending

The dispersion is defined by the relation:

$$D_{\sigma_F} = \overline{\sigma_F^2} \cdot c_{v\sigma_F}^2 \tag{2}$$

where $c_{v\sigma_F}$ represents the variation coefficient of the tension due to tiredness.

This coefficient can be calculated using the algebra elements of random variables.

The dispersion specific to limit tension due to tiredness is calculated according to the formula below:

$$D_{\sigma_{Flim}} = \overline{\sigma_{Flim}^2} \cdot c_{v\sigma_{Flim}}^2 \tag{3}$$

The $c_{v\sigma_{Flim}}$ variation coefficient can be calculated according to the specialty literature.

When σ_F represents the effective tension specific to a tooth within a gearing, the breaking probability of the tooth after a t_f period of time is calculated as:

$$F_f = \int_0^{t_F} f_z(t)dt \qquad (4)$$

where f_z is the frequency function or the breaking distribution of the teeth (fig. 1). The f_z function can be correlated both with the limit tension and the effective tension.

Considering $z_F = \sigma_F - \sigma_{Flim}$, for the normal distribution of the limit tension and of the effective tension relation no. 4 can be written as:

$$F_F = \frac{1}{\sqrt{D_{\sigma_{Flim}} + D_{\sigma_F}} \times \sqrt{2p}} \cdot \int_0^{\infty} e^{-\frac{\left[z_F - (\overline{\sigma_r} - \overline{\sigma_{Flim}})^2\right]}{2\left(D_{\sigma_{Flim}} + D_{\sigma_F}\right)}} dz_F = 0{,}5 - \left(\frac{1}{\sqrt{2p}} \cdot \int_0^{a_F} e^{\frac{-x^2}{2}} dx\right) = 1 - f(a_F) = 1 - \frac{\gamma_F}{100} \qquad (5)$$

where:

$$x = \frac{z_F - (\overline{\sigma_F} - \overline{\sigma_{Flim}})}{\left(D_{\sigma_{Flim}} + D_{\sigma_F}\right)^{\frac{1}{2}}}$$

g_F - the percentage reliability

$f(a_F)$ - Laplace function, with a_f argument

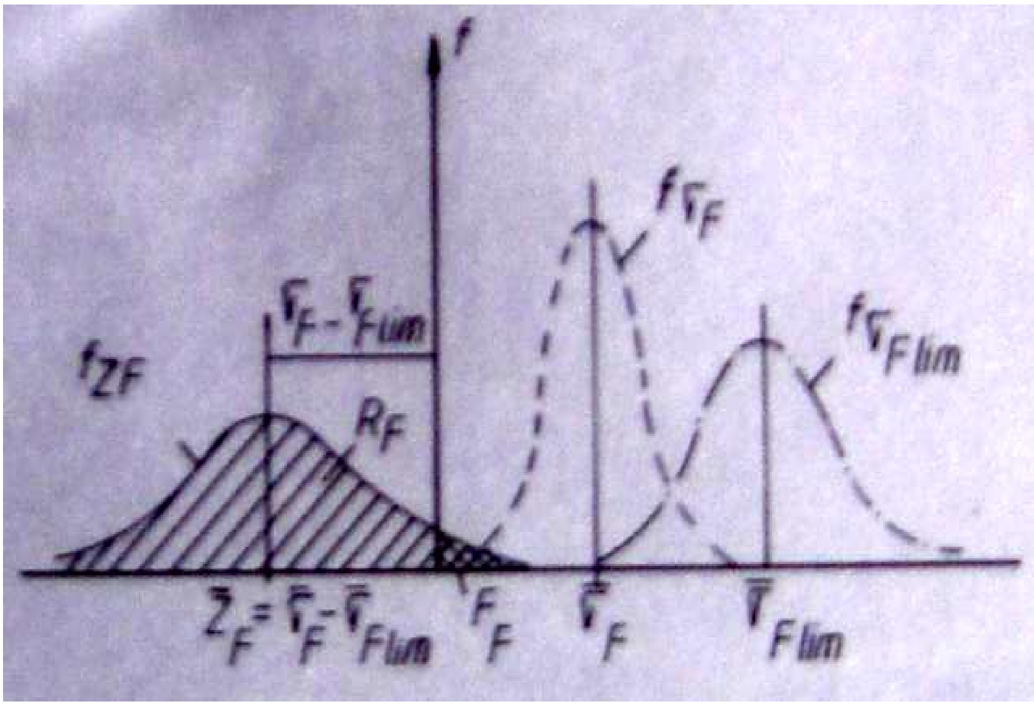

Figure no. 1 – The statistic safety parameter of the teeth

Depending on the N number of cycles and on the K_{FL} factor of the cycles number the probability of operating without deterioration is calculated as follows:

$$R_F(N) = 1 - \int_{-\infty}^{m\frac{s_F - K_{FL}}{s_R}} \exp\left(-\frac{x^2}{2}\right) dx \qquad (6)$$

where σ_R represents the medium square deviation of the deteriorations distribution curve.

Considering the setting of output rates it results:

$$m\frac{s_F - K_{FL}}{\sigma_R} = \pm 1 \qquad (7)$$

Relations no. 6 and 7 are using the s_F variable which represents the reversed of the safety bending breaking factor.

In this respect we can write: $s_F = \overline{\sigma_F} / \overline{s_{FlimD}}$, $\overline{\sigma_{FlimD}} = \overline{\sigma_{Flim}} / \overline{K_{FL}}$, where $\overline{K_{FL}}$ represents the medium value of the K_{FL} number of cycles factor.

Taking into consideration relation no. 3 the a_f statistic safety parameter of the teeth can be written:

$$a_F = \frac{\overline{K_{FL}} - s_F}{\left(\overline{K_{FL}}^2 \cdot c^2_{vs_{Flim}} + s_F^2 c^2_{vs_F}\right)^{\frac{1}{2}}} \qquad (8)$$

Using relations no. 7 and 8, not taking into consideration the $c^2_{vs_{Flim}} \cdot c^2_{vs_F}$ it results the medium square deviation for the left and for the right branch of the curve:

$$\sigma_{R_1} = m \cdot \left(-c^2_{v\sigma_F} + \sqrt{c^2_{v\sigma_{Flim}} + c^2_{v\sigma_F}}\right), \text{for } N \leq \overline{N}$$

$$\sigma_{R_2} = m \cdot \left(c^2_{v\sigma_F} + \sqrt{c^2_{v\sigma_{Flim}} + c^2_{v\sigma_F}}\right), \text{for } N > \overline{N}$$

The symmetry condition is: $c_{vs_F} = c_{vs_F} \cdot \left(1 - c^2_{vs_F}\right)^{\frac{1}{2}}$

The distribution curve can be considered symmetric if the variation coefficients do not differ more than 25%.

Using $c_F = \frac{s_F}{K_{FL}}$ and not taking into consideration the $a_F^4 c^2_{vs_F} c^2_{vs_{F\lim}}$ term in relation no. 8, it results:

$$c_F = \frac{1 - a_F\sqrt{c^2_{vs_{F\lim}} + c^2_{vs_F}}}{1 - a_F^2 c^2_{vs_F}} \qquad (9)$$

Accepting the g_F percentage reliability used in relation no. 5, we can determine a_f. Using relation no. 9 there can be calculated c_f *and* K_{FL}. Further on there can be determined the *N* number of operating cycles and the t_h operating period of time.

Tabel no. 1 contains the valuation of reliability depending on its value and the value of the a_f statistic parameter.

Table no. 1

Teeth reliability R_f	**Value**	**Statistic parameter** a_f
0.99	Small	2.33
0.999	Medium	3.10
0.9999	Big	3.72

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